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GRAVITATIONAL INTERACTION AND POINCARÉ GAUGE THEORY OF GRAVITY

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Some principal problems of general relativity theory and attempts of their solution are discussed. The Poincaré gauge theory of gravity as natural generalization of Einsteinian gravitation theory is considered. The changes of gravitational interaction in the frame of this theory leading to the solution of principal problems of general relativity theory are analyzed.

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1. Introduction

Einsteinian general relativity theory (GR) is the base of modern theory of gravitational interaction, relativistic cosmology and astrophysics. GR allows to describe different gravitating systems and cosmological models at widely changing scales of physical parameters. At the same time GR possesses certain principal difficulties, which, in particular, appear in cosmology. One of the most principal cosmological problems remains the problem of cosmological singularity (PCS): various cosmological models describing the evolution of the Universe have the beginning in time in the past and in accordance with Einstein gravitation equations the singular state with divergent energy density and singular metrics appear at the beginning of cosmological expansion. The PCS is a particular case of general problem of gravitational singularities of GR appearing by description of gravitating systems at extreme conditions (extremely high energy densities and pressures) [1]. Other principal problem of GR is connected with explanation of cosmological and astrophysical observations. To explain observational cosmological and astrophysical data in the framework of GR it is necessary to suppose that approximately 96%

energy in the Universe is related to some hypothetical kinds of gravitating matter — dark energy and non-baryonic dark matter, and only 4% energy is related to usual gravitating matter, from which galaxies are built. As a result, the situation in cosmology and generally in gravitation theory actually in certain relation is similar to that in physics at the beginning of XX century, when the notion of "ether" was introduced in order to explain various electrodynamic phenomena. As it is well known, the creation of special relativity theory by A. Einstein allowed to solve existed problems without "ether" notion.

There were many attempts with the purpose to solve indicated problems of GR. We will discuss briefly the most known from such attempts. Because in the frame of GR there are not restrictions on admissible values of energy density, and the energy density can reache the Planckian scale, according to opinion of many physicists the solution of PCS has to be connected with quantum gravitation theory. A number of regular cosmological solutions was obtained in the frame of candidates to quantum gravitation theory — string theory/M-theory and loop quantum gravity [2–5]. Radical ideas connected with notions of strings, branes, extra-dimensions, space-time foam *etc.* are used in these works. Note that indicated works are not free from some problems and difficulties. So, the obtaining of regular cosmological solutions in the frame of string theory is connected with the breakdown of physical condition of energy density positivity for gravitating matter (see, for example, [2, 3]). Moreover, the most part of cosmological solutions in string theory by transition to 4-dimensional system of reference are singular [6]. In connection with this note that the solution of PCS, from our point of view, means not only obtaining regular cosmological solutions, but also excluding singular solutions, this means all physically acceptable cosmological solutions (or the most part of such solutions) in the frame of correct gravitation theory have to be regular. Bouncing cosmological solutions obtained in loop quantum gravity [4, 5] by more exact calculations contain gravitational singularity with divergent Hubble parameter [7]. The dark energy or quintessence as hypothetic kind of gravitating matter with negative pressure was introduced with the purpose to explain accelerating cosmological expansion at present epoch in the frame of GR [8]. In many papers the dark energy is related to vacuum energy leading to cosmological constant in Einstein gravitation equations. By taking into account that the vacuum energy density has divergent value in quantum field theory and can be eliminated by means of regularization procedure, the following question appears: why only the very small part of it is manifested as cosmological constant? Unlike dark energy, the distribution of dark matter in space is not homogeneous; by using the method of gravitational lensing, dark matter maps of its distribution in the Universe were made [9]. The nature of dark matter is unknown yet; according to the opinion of many physicists the dark matter is formed from weak interacting massive particles (WIMP) revealed in united supersymmetric models of elementary particles.

There is other treatment with the purpose of the solution of discussed problems by using non-Einsteinian theories of gravity. So, now one discusses largely gravitation theory in Riemannian space-time, based on gravitational Lagrangian in the form of some function of a scalar curvature with the purpose to solve the dark energy problem of GR (see, for example, [10]). The phenomenological change of Newton's dynamics was considered in so-called MOND in order to solve the dark matter problem [11]. In the frame of different non-Einsteinian theories of gravity the PCS was also studied. We do not have an aim to consider here all these theories. Note only that the most part of various generalizations of Einsteinian theory of gravitation do not have solid theoretical foundation.

At the same time now there is the gravitation theory built in the framework of common field-theoretical approach including the local gauge invariance principle in 4-dimensional physical space-time, which is a natural generalization of GR and which offers opportunities to solve its principal problems. It is the Poincaré gauge theory of gravity (PGTG). The main goal of this paper is to attract attention to this fact. In Section 2 the question "Why we need the PGTG?" is considered. In Section 3 the changes of law of gravitational interaction by certain physical conditions in the frame of PGTG and their physical consequences are discussed.

2. Why we need the Poincaré gauge theory of gravity?

As it is known, the local gauge invariance principle is the basis of modern theories of fundamental physical interactions. From the physical point of view, this principle establishes the correspondence between certain important conserving physical quantities, connected according to the Noether theorem with some symmetries groups, and fundamental physical fields, which have as a source corresponding physical quantities and play the role of carriers of fundamental physical interactions. The applying of this principle to gravitational interaction leads, generally speaking, to generalization of Einsteinian theory of gravitation. Note that because the sources of gravitational field are connected with space-time transformations, the gauge treatment to gravitation has some differences in comparison with Yang–Mills fields connected with internal symmetries groups.

GR and generally metric theories of gravity, in the frame of which the energy-momentum tensor plays the role of source of gravitational field, can be introduced by localizing the 4-parametric translations group [12,13]. Because the localized translations group leads us to general coordinate transformations, from this point of view the general covariance of GR is connected with gauge approach. At the same time the local Lorentz group (group of tetrad Lorentz transformations) in GR and other metric theories of gravitation does not play any dynamical role from the point of view of gauge approach, because the corresponding Noether invariant in these theories is identically equal to zero [14]. The including of the tetrad Lorentz group to gravitational gauge group leads to the PGTG [15] (see also *e.g.* [16] and references herein). In the frame of PGTG the gauge Lorentz field, which has transformation properties of anholonomic Lorentz connection [17], is considered together with orthonormal tetrad as independent gravitational field variables, as a result the PGTG is gravitation theory in the 4-dimensional Riemann–Cartan space-time U_4. By other words, if one means that the Lorentz group, which is fundamental group in physics, plays the dynamical role in the gauge field theory and the Lorentz gauge field exists in the nature, we obtain necessarily the gravitation theory in the Riemann-Cartan space-time as natural generalization of GR. The torsion tensor $S^i{}_{\mu\nu}$ and the curvature tensor $F^{ik}{}_{\mu\nu}$ play the role of gravitational field strengths in PGTG and are defined by the tetrad $h^i{}_\mu$ and the Lorentz connection $A^{ik}{}_\mu$ by the following way:

$$S^i{}_{\mu\,\nu} = \partial_{[\nu}\, h^i{}_{\mu]} - h_{k[\mu}A^{ik}{}_{\nu]}\,, \quad F^{ik}{}_{\mu\nu} = 2\partial_{[\mu}A^{ik}{}_{\nu]} + 2A^{il}{}_{[\mu}A^k{}_{|l\,|\nu]}\,, \tag{1}$$

where holonomic and anholonomic components are denoted by means of Greek and Latin indices respectively. If one uses minimal coupling of matter with gravitational field, the energy-momentum and spin tensors play the role of sources of gravitational field in PGTG. Unlike gauge Yang–Mills fields, the translational gauge strength — the torsion tensor defined by (1) — depends also on the Lorentz gauge field. As a result in general case the torsion can be created by the energy-momentum as well as by the spin momentum of gravitating matter. Despite the opinion presented in literature (see, for example, [18]) that the torsion (non-Riemannian space-time characteristics) is essential only for gravitating matter having the spin momentum, really the torsion can play the principal role in the case of usual spinless gravitating systems (see below Section 3). Although the direct interaction of the torsion with minimally coupled spinless matter is absent, the dynamics of such gravitating systems depends essentially on space-time torsion by virtue of the interaction between metric and torsion fields. Note also that the correspondence between conserving physical quantities and gauge fields discussed at the beginning of this Section has some peculiarity in the case of PGTG. The law of conservation of angular momentum current in the frame of the field theory in Minkowsky space-time includes together with the spin also orbital part. The procedure leading to this conservation law is realized by using the system of orthogonal cartesian coordinates, where the coordinate basis and the local Lorentz tetrad coincide, and the origin of orbital part of angular momentum current is connected with coordinate transformations essentially and its tensor definition is possible only in this case. However, the orbital part of angular momentum current is not a source of gravitational field in PGTG; really it cannot play this role in the frame of not homogeneous space-time, where its tensor definition does not exist (*cf.* [19]). Note also that only localized tensor quantities can be sources of physical fields. Unlike spin momentum the angular orbital momentum does not have this property: even in Minkowsky space-time, where tensor definition of angular orbital momentum is possible, its value depends on the choice of the origin of coordinates system. We will consider below the PGTG as relativistic gravitation theory in 4-dimensional Riemann-Cartan space-time, in the frame of which the gravitational field is described by means of interacting metric and torsion fields and is created by energy-momentum and spin tensors of gravitating matter.

The structure of gravitational equations of PGTG depends on the choice of gravitational Lagrangian $\mathcal{L}_\mathrm{g}$ built by means of gravitational field strengths (and tetrad or metrics). The simplest PGTG is the Einstein–Cartan theory, which corresponds to the choice of $\mathcal{L}_\mathrm{g}$ in the form of scalar curvature of U_4 and in the frame of which the torsion and spin tensors are connected by linear algebraic way [15,20]. The Einstein–Cartan theory is a degenerate theory; in particular, in the frame of this theory the torsion is equal to zero identically for spinless matter, although the torsion tensor, as it was noted above, is gravitational strength corresponding to translations connected with energy-momentum tensor directly. Similar to Yang–Mills fields the gravitational Lagrangian of PGTG has to include terms quadratic in gravitational field strengths. Because explicit form of quadratic part of $\mathcal{L}_\mathrm{g}$ is unknown, we will consider PGTG by choosing $\mathcal{L}_\mathrm{g}$ in general form including various invariants quadratic in the curvature and torsion tensors:

$$\begin{aligned}\mathcal{L}_\mathrm{g} \;=\; & f_0\, F + F^{\alpha\beta\mu\nu}\left(f_1\, F_{\alpha\beta\mu\nu} + f_2\, F_{\alpha\mu\beta\nu} + f_3\, F_{\mu\nu\alpha\beta}\right)\\ & + F^{\mu\nu}\left(f_4\, F_{\mu\nu} + f_5\, F_{\nu\mu}\right) + f_6\, F^2\\ & + S^{\alpha\mu\nu}\left(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}\right) + a_3\, S^{\alpha}{}_{\mu\alpha} S_{\beta}{}^{\mu\beta}\,, \qquad (2)\end{aligned}$$

where $F_{\mu\nu} = F^{\alpha}{}_{\mu\alpha\nu}$, $F = F^{\mu}{}_{\mu}$, f_i $(i = 1, 2, \ldots, 6)$, a_k $(k = 1, 2, 3)$ are indefinite parameters, $f_0 = (16\pi G)^{-1}$, G is Newton's gravitational constant (the light speed in the vacuum $c = 1$). Gravitational equations of PGTG are deduced from the action integral $I = \int \left(\mathcal{L}_g + \mathcal{L}_m\right) h d^4x$, where $h = \det\left(h^i{}_\mu\right)$ and $\mathcal{L}_m$ is the Lagrangian of gravitating matter. Although the gravitational Lagrangian (2) includes a number of indefinite parameters, gravitational equations of PGTG for homogeneous isotropic models (HIM) considered below depend weakly on the choice of quadratic part of gravitational Lagrangian by virtue of their high spatial symmetry. The investigation of HIM in the frame of PGTG leads to some important physical consequences concerning the gravitational interaction for usual gravitating matter.

3. On gravitational interaction in PGTG

The system of gravitational equations of PGTG corresponding to gravitational Lagrangian (2) in general case is a complicated system of differential equations. From mathematical point of view, HIM are the most simple models, for which the system of gravitational equations of PGTG has a sufficiently simple form. From a physical point of view, the study of HIM has important cosmological applications. Isotropic cosmology including inflationary cosmology in the frame of PGTG was built and investigated in our works (see [21–26] and references herein). Now we will discuss some physical consequences of general character following from investigation of HIM. The dynamics of HIM in PGTG is described in general case by three functions of time: the scale factor of Robertson-Walker metrics $R(t)$ and two torsion functions $S_1(t)$ and $S_2(t)$ determining the form of the torsion tensor. The functions $S_1(t)$ and $S_2(t)$ have different properties with respect to transformations of spacial inversions: unlike $S_1(t)$ the function $S_2(t)$ has pseudoscalar character. We will consider two types of HIM: HIM with one torsion function S_1 ($S_2 = 0$) and HIM with two torsion functions.

At first, we will consider HIM with vanishing pseudoscalar torsion function [21–23] filled by gravitating matter with energy density ρ and pressure p (the average of spin distribution is equal to zero). In this case gravitational equations of PGTG lead to the following generalized cosmological Friedmann equations (GCFE):

$$\frac{k}{R^2}+\left\{\frac{d}{dt}\ln\left[R\sqrt{|1+\alpha\,(\rho-3p)|}\right]\right\}^2=\frac{8\pi G}{3}\,\frac{\rho+\frac{\alpha}{4}\,(\rho-3p)^2}{1+\alpha\,(\rho-3p)}\,, \tag{3}$$

$$R^{-1}\,\frac{d}{dt}\left[\frac{dR}{dt}+R\frac{d}{dt}\left(\ln\sqrt{|1+\alpha\,(\rho-3p)|}\right)\right]=-\frac{4\pi G}{3}\,\frac{\rho+3p-\frac{\alpha}{2}\,(\rho-3p)^2}{1+\alpha\,(\rho-3p)}\,, \tag{4}$$

where indefinite parameter $\alpha=\dfrac{f}{3{f_0}^2}>0$ ($f=f_1+\frac{f_2}{2}+f_3+f_4+f_5+3f_6$) has the inverse dimension of energy density. The GCFE are obtained by using the expression for the torsion function $S_1(t)$ following from gravitational equations in the form:

$$S_1=-\frac{1}{4}\frac{d}{dt}\ln|1+\alpha(\rho-3p)|\,. \tag{5}$$

Because the equations of motion for spinless matter in the frame of PGTG (by minimal coupling with gravitation) have the same form as in GR, the conservation law for gravitating matter has usual form:

$$\dot{\rho} + 3H\,(\rho + p) = 0\,, \tag{6}$$

where $H = \dot{R}/R$ is the Hubble parameter and a dot denotes the differentiation with respect to time[1]. The difference of (3)–(4) from Friedmann cosmological equations of GR is connected with terms containing the parameter α. The value of α^{-1} determines the scale of extremely high energy densities. Solutions of GCFE coincide practically with corresponding solutions of GR, if the energy density is small $|\alpha(\rho - 3p)| \ll 1$ ($p \neq \frac{1}{3}\rho$). The difference between GR and PGTG can be significant at extremely high energy densities $|\alpha(\rho - 3p)| \sim 1$, where the dynamics of HIM depends essentially on space-time torsion.

The structure of GCFE (3)–(4) ensures regular behavior of cosmological solutions. It is because the gravitational interaction at extreme conditions changes and has the repulsive character [22]. Unlike the gravitational repulsion effect in the frame of Einstein–Cartan theory, which appears for spinning matter [27] and critically depends on the spin description [28], in the case of discussed HIM the gravitational repulsion takes place for usual spinless matter that leads to regularity of ordinary cosmological HIM. In order to demonstrate this fact in the case of inflationary cosmological models, we will consider HIM filled with scalar field ϕ minimally coupled with gravitation and gravitating matter with equation of state in the form $p_m = p_m(\rho_m)$ (values of gravitating matter are denoted by means of index "m"). Then the energy density ρ and the pressure p take the form

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + V + \rho_m \qquad (\rho > 0)\,, \qquad p = \tfrac{1}{2}\dot{\phi}^2 - V + p_m\,, \tag{7}$$

where $V = V(\phi)$ is a scalar field potential. Because the energy density ρ is positive and $\alpha > 0$, from equation (3) in the case $k = +1$, 0 follows the relation:

$$Z = 1 + \alpha\,(\rho - 3p) = 1 + \alpha\left(4V - \dot{\phi}^2 + \rho_{\mathrm{m}} - 3p_{\mathrm{m}}\right) \geq 0\,. \tag{8}$$

The relation (8) is valid also for cosmological solutions of open type ($k = -1$) [21]. The domain of admissible values of scalar field ϕ, time derivative $\dot{\phi}$ and energy density ρ_m determined by (8) is limited in space P of these variables by bound L defined as

$$Z = 0 \qquad \text{or} \qquad \dot{\phi} = \pm\left(4V + \alpha^{-1} + \rho_{\mathrm{m}} - 3p_{\mathrm{m}}\right)^{1/2}\,. \tag{9}$$

[1] It is easy to show that the conservation law (6) follows directly from (3)–(4).

Unlike GR at compression stage the time derivative $\dot{\phi}$ does not diverge, and by reaching the bound L the transition to the second part of cosmological solution containing the expansion stage takes place. From cosmological equation (3) by using the conservation law (6) follows that in space P there are extremum surfaces, in points of which the Hubble parameter vanishes [21, 23]. Extremum surfaces play the role of "bounce surfaces", because the time derivative of the Hubble parameter is positive on the greatest part of these surfaces in the case of scalar field potentials applying in chaotic inflation. All cosmological solutions have bouncing character and are regular with respect to metrics, Hubble parameter and its time derivative. If gravitating matter satisfies standard conditions (energy density is positive, energy dominance condition is valid), any cosmological solution is not limited in the time, and before the expansion stage cosmological solution contains the compression stage and regular transition from compression to expansion. Note that the character of gravitational interaction in the frame of PGTG depends essentially on properties of gravitating matter and at, first of all, on its equation of state. The effect of gravitational repulsion at extreme conditions in considered HIM takes place by virtue of the following condition for total energy density and pressure: $p > \frac{1}{3}\rho$. In the case of inflationary HIM including together with usual gravitating matter also scalar fields this condition is realized at certain moment of compression stage always independently on conditions for usual matter: $p_m = \frac{1}{3}\rho_m$ or $p_m > \frac{1}{3}\rho_m$. For gravitating matter the condition $p_m > \frac{1}{3}\rho_m$ is valid also at sufficiently high energy densities [29]. The HIM filled with such a matter without scalar fields have the limiting energy density, which is reaching at a bounce and is determined from the relation $Z = 0$ [30]. During the expansion stage, when the energy density becomes sufficiently small and the equation of state changes ($p < \frac{1}{3}\rho$), the GCFE lead to additional gravitational attraction in comparison with GR and Newton's theory of gravity. In particular, at matter dominating stage with equation of state for dust ($p = 0$), by taking into account the relation $(\alpha\rho) \ll 1$, it is easy to obtain from (3)–(4) in the case $k = 0$ that

$$\ddot{R}/R = -\frac{4\pi G}{3}\rho(1 + 9\alpha\rho)\,. \tag{10}$$

According to (10) the force of gravitational attraction is $(1 + 9\alpha\rho)$ times bigger than in Newton's theory of gravity.

The space-time torsion in PGTG can lead to gravitational repulsion effect not only at extreme conditions, but also at very small energy densities. Such situation takes place in the case of HIM with two torsion functions [24, 26]. Cosmological equations for such HIM include also the pseudoscalar torsion function S_2 with its first time derivative and contain besides α two others indefinite parameters: $b = a_2 - a_1$ with dimension of parameter f_0 and dimensionless parameter ε, which is the function of coefficients f_i. If $|\varepsilon| \ll 1$, the pseudoscalar torsion function contains at asymptotics, where physical parameters of cosmological model are sufficiently small, some not vanishing value and is equal to:

$$S_2^2 = \frac{1}{12b}\left[\rho - 3p + \alpha^{-1}(1 - b/f_0)\right] . \tag{11}$$

As a result cosmological equations at asymptotics take the form of cosmological Friedmann equations with effective cosmological constant induced by pseudoscalar torsion function:

$$\frac{k}{R^2} + H^2 = \frac{1}{6b}\left[\rho + \frac{1}{4}\alpha^{-1}(1 - b/f_0)^2\right] , \tag{12}$$

$$\dot{H} + H^2 = -\frac{1}{12b}\left[\rho + 3p - \frac{1}{2}\alpha^{-1}(1 - b/f_0)^2\right] . \tag{13}$$

By using at asymptotics the equation of state for dust ($p = 0$), we see that cosmological equations (12)–(13) lead to observable accelerating cosmological expansion by certain relation between indefinite parameters b and α. If we suppose that the scale of extremely high energy densities defined by α^{-1} is larger than the energy density for quark–gluon matter, but less than the Planckian one, then we can obtain the corresponding estimation for b, which is very close to f_0. Hence, the acceleration of cosmological expansion in PGTG has geometrical nature and is connected with the change of gravitational interaction induced by space-time torsion. The investigation of inflationary HIM with two torsion functions at extreme conditions at the beginning of cosmological expansion shows that the PGTG allows to build totally regular inflationary Big Bang scenario by classical description of gravitational field. If the energy density and values of torsion functions at transition stage from compression to expansion are less than the Planckian ones, quantum gravitational era was absent by evolution of the Universe. If the Planckian conditions were realized at the beginning of cosmological expansion, quantum gravitational corrections have to be taken into account; however, classical cosmological equations of PGTG ensure the regular character of the Universe evolution.

4. Conclusion

From our consideration given above follows that the PGTG can have the important meaning for theory of gravitational interaction. The PGTG leads to certain principal differences in comparison with GR concerning the character of gravitational interaction for usual spinless gravitating matter.

According to PGTG, the domain of applicability of GR is limited, namely in the case of cosmological HIM the domain of admissible energy densities has upper limit determined by α^{-1} and lower limit equal to $\frac{1}{4}\alpha^{-1}(1-b/f_0)^2$. The investigation of HIM with two torsion functions shows that the torsion can be important in Newtonian approximation (see (11)) and the Newton's law of gravitational attraction has limits of its applicability in the case of usual gravitating systems with sufficiently small energy densities. As it was noted above, the law of gravitational interaction for such gravitating systems can include corrections corresponding to additional attraction. This means that the investigation of not homogeneous gravitating systems at galactic scales, in particular, of spherically-symmetric systems in the frame of PGTG is of direct physical interest in connection with the problem of dark matter of GR with the purpose of its solution.

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