De Sitter spacetime with torsion as physical spacetime in the vacuum and isotropic cosmology

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Homogeneous isotropic models with two torsion functions built in the framework of the Poincaré gauge theory of gravity (PGTG) based on general expression of gravitational Lagrangian without cosmological constant are analyzed. It is shown that the physical spacetime in the vacuum in the frame of PGTG can have the structure of flat de Sitter spacetime with torsion. Some physical consequences of obtained conclusion are discussed.

Keywords: De Sitter spacetime, torsion, gauge theory of gravity, isotropic cosmology

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1. Introduction

Since the creation of the special relativity theory the physical spacetime in the vacuum (without physical fields) is considered as Minkowski spacetime with the structure of pseudo-Euclidien continuum. According to the general relativity theory (GR), the physical spacetime in the gravitational field possesses the structure of pseudo-Riemannien continuum, however, far from gravitating objects and in absence of gravitational waves the physical spacetime in GR in fact can be considered as Minkowski spacetime [a]. In the framework of the Poincaré gauge theory of gravity [1,2,3,4,5,6], which is a natural and in certain sense necessary generalization of Einsteinian GR (see Refs. 7–8 and references herein), the physical spacetime in the gravitational field has the structure of Riemann-Cartan continuum. Usually one supposes that in the frame of PGTG, similarly to GR, far from gravitating objects and in absence of gravitational waves the properties of physical spacetime are practically the same that of Minkowski spacetime. However, as it will be shown in

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[a]Here and later we assume that cosmological constant is equal to zero.

this paper, the situation in PGTG can be essentially different, and the structure of physical spacetime in the vacuum can vary from that of Minkowski spacetime which leads to important physical consequences.

Before we analyze the corresponding situation in the framework of PGTG, it is necessary to note why this is of direct physical interest for the gravitation theory, modern cosmology and astrophysics. As it was shown in a number of papers (see Refs. 9–15 and references herein), the PGTG offers opportunities to solve principal cosmological problems – the problem of cosmological singularity, the problem of dark components of the Universe – dark energy and dark matter by describing the gravitational field in 4-dimensional classical physical space-time. It is because the PGTG leads to essential changes of gravitational interaction in comparison with GR and Newton's theory of gravity by certain physical conditions, in particular at extreme conditions (extremely high energy densities and pressures) in the beginning of cosmological expansion. These changes are connected with the structure of physical spacetime in PGTG, namely with spacetime torsion, the presence of which is a necessary consequence of including the Lorentz group to the gauge group which corresponds to gravitational interaction.

The present paper is organized in the following way. In Section 2 we present the main relations for homogeneous isotropic models (HIM) obtained in the framework of PGTG based on general expression of gravitational Lagrangian. In Section 3 the structure of spacetime in the vacuum in PGTG is analyzed. In Conclusion some physical consequences connected with the structure of physical spacetime in the vacuum are discussed.

2. Homogeneous isotropic models in PGTG

From the physical point of view the spacetime in the vacuum is homogeneous and isotropic, and in order to investigate the structure of vacuum spacetime in the frame of PGTG we will analyze the HIM. We will consider the PGTG based on general expression of gravitational Lagrangian $\mathcal{L}_\mathrm{g}$ including both a scalar curvature and various invariants quadratic in gravitational gauge field strengths – the curvature and torsion tensors (definitions and notations of [14] are used below):

$$\mathcal{L}_\mathrm{g} = f_0\,F + F^{\alpha\beta\mu\nu}(f_1\,F_{\alpha\beta\mu\nu} + f_2\,F_{\alpha\mu\beta\nu} + f_3\,F_{\mu\nu\alpha\beta}) + F^{\mu\nu}(f_4\,F_{\mu\nu}$$
$$+ f_5\,F_{\nu\mu}) + f_6\,F^2 + S^{\alpha\mu\nu}(a_1\,S_{\alpha\mu\nu} + a_2\,S_{\nu\mu\alpha}) + a_3\,S^\alpha{}_{\mu\alpha}S_\beta{}^{\mu\beta}. \qquad (1)$$

The Lagrangian (1) includes the parameter $f_0 = (16\pi G)^{-1}$ (G is Newton's gravitational constant, the light velocity $c = 1$) and a number of indefinite parameters: f_i $(i = 1, 2, ...6)$ and a_k $(k = 1, 2, 3)$. By using the expression (1) for $\mathcal{L}_\mathrm{g}$ the system of gravitational equations for HIM filled by gravitating matter with energy density ρ and pressure p was obtained in [12] (see also [14]). This system contains 4 differential equations for three geometric characteristics of HIM as functions of time – the scale

factor of Robertson-Walker metrics R and two torsion functions S_1 and S_2 [b]. Generally the system of gravitational equations for HIM contains 5 following indefinite parameters:

$$a = 2a_1 + a_2 + 3a_3, \qquad b = a_2 - a_1, \qquad f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6,$$

$$q_1 = f_2 - 2f_3 + f_4 + f_5 + 6f_6, \qquad q_2 = 2f_1 - f_2.$$

Gravitational equations for HIM allow to obtain cosmological equations generalizing Friedmann cosmological equations of GR in the following form:

$$\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 = A_1, \tag{2}$$

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = A_2, \tag{3}$$

where the curvature functions A_1 and A_2 are determined from gravitational equations by the following way:

$$A_1 = -\frac{1}{12(f_0 + a/8)Z}\Big[\rho + 3p - \frac{2f}{3}F^2 + 8q_2 F S_2^2$$

$$-12q_2\left(\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right) - \frac{3a}{2}\left(\dot{H} + H^2\right)\Big],$$

$$A_2 = \frac{1}{6(f_0 + a/8)Z}\Big[\rho - 6(b + a/8)S_2^2 + \frac{f}{3}F^2 + \frac{3a}{4}\left(\frac{k}{R^2} + H^2\right)$$

$$-6q_2\left(\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right)\Big], \tag{4}$$

$H = \dot{R}/R$ is the Hubble parameter (a dot denotes the differentiation with respect to time), the scalar curvature $F = 6(A_1 + A_2)$ is

$$F = \frac{1}{2(f_0 + a/8)}\Big[\rho - 3p - 12(b + a/8)S_2^2 + \frac{3a}{2}\left(\frac{k}{R^2} + \dot{H} + 2H^2\right)\Big], \tag{5}$$

and $Z = 1 + \frac{1}{(f_0 + a/8)}\left(\frac{2f}{3}F - 4q_2 S_2^2\right)$. Cosmological equations (2)-(3) contain the torsion functions S_1 and S_2 with their first derivatives. From gravitational equations for HIM the torsion function S_1 is determined as

$$S_1 = -\frac{1}{6(f_0 + a/8)Z}\left[f\dot{F} + 6(2f - q_1 + 2q_2)HS_2^2 + 6(2f - q_1)S_2\dot{S}_2\right], \tag{6}$$

and the torsion function S_2 satisfies the differential equation of the second order:

$$q_2\left[\ddot{S}_2 + 3H\dot{S}_2 + \left(3\dot{H} - 4\dot{S}_1 + 4S_1(3H - 4S_1)\right)S_2\right]$$

$$-\left[\frac{q_1 + q_2}{3}F + (f_0 - b) - 2(q_1 + q_2 - 2f)A_2\right]S_2 = 0. \tag{7}$$

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[b]At the first time equations for HIM with two torsion functions were deduced in Ref. 16. These equations were considered in [17] with the purpose to obtain their solutions; however, so called "modified double duality ansatz" used in [17] by obtaining solutions with not vanishing torsion function S_2 is not applicable in this case even for the vacuum (see below) and its application leads generally speaking to incorrect solutions.

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By given equation of state for gravitating matter cosmological equations (2)-(3) together with equations (6)-(7) for torsion functions describe the evolution of the most general HIM in PGTG. So far, we have not used any restrictions on indefinite parameters of $\mathcal{L}_g$. From formulas (5)-(6) for scalar curvature F and torsion function S_1 we see that cosmological equations (2)-(3) do not contain higher derivatives of the scale factor R only if $a = 0$. Isotropic cosmology with $a \neq 0$ possesses some principal problems; in particular cosmological equations at physically available initial conditions lead in this case to not physical solutions[24]. With the purpose to exclude higher derivatives of R from cosmological equations the restriction $a = 0$ is used in our works. Because of mathematical reasons this restriction will be not used by further general mathematical analysis.

3. Spacetime of gravitating vacuum in PGTG

Apart the spacial homogeneity and isotropy, in order to investigate the gravitating vacuum in PGTG we have to take into account also its homogeneity in time. In the frame of GR these conditions in accordance with Friedmann cosmological equations (in absence of gravitating matter ($\rho = 0$) and vanishing cosmological constant) are fulfilled only in the case of flat HIM ($k = 0$) with zeroth curvature that leads to Minkowski spacetime [c]. However, in the frame of PGTG there is also another solution, which can be obtained if we suppose that in relations (2)-(7) for HIM the time derivatives vanish and $\rho = 0$. Then from (5) and (6) we obtain the following expressions for scalar curvature F and torsion function S_1 in the vacuum ($k = 0$):

$$F = \frac{6}{f_0 + a/8}\left[-(b + a/8)S_2^2 + \frac{a}{4}H^2\right],$$
$$S_1 = -\frac{2f - q_1 + 2q_2}{(f_0 + a/8)Z}HS_2^2, \tag{8}$$

and the curvature functions (4) in the vacuum take the following form:

$$A_1 = \frac{1}{6(f_0 + a/8)Z}\left[\frac{f}{3}F^2 - 4q_2FS_2^2 + 6q_2\left(H^2 - 4S_2^2\right)S_2^2 + \frac{3a}{4}H^2\right],$$
$$A_2 = \frac{1}{6(f_0 + a/8)Z}\left[-6(b + a/8)S_2^2 + \frac{f}{3}F^2 + \frac{3a}{4}H^2 - 6q_2\left(H^2 - 4S_2^2\right)S_2^2\right]. \tag{9}$$

Then cosmological equations (2)-(3) in the vacuum take the following form:

$$H^2\left[1 + \frac{2(2f - q_1 + 2q_2)}{(f_0 + a/8)Z}S_2^2\right] = A_1,$$
$$H^2\left[1 + \frac{2(2f - q_1 + 2q_2)}{(f_0 + a/8)Z}S_2^2\right]^2 - S_2^2 = A_2. \tag{10}$$

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[c]Such model filled with the dust ($p = 0$) was considered in [18] as preferable model of the real universe.

Because the Bianchi identity for HIM [12] leads in this particular case to the following relation: $H(A_2 - A_1) + 2S_1A_1 + HS_2^2 = 0$, only one out of eqs. (10) is independent. Eq. (7) for S_2-function in the vacuum is transformed to:

$$S_2[4q_2S_1(3H - 4S_1) - \frac{q_1 + q_2}{3}F + 2(q_1 + q_2 - 2f)A_2 - (f_0 - b)] = 0, \quad (11)$$

where the curvature functions and S_1-function are determined by (8)-(9). Eqs. (10)-(11) determine the values of H and S_2 for gravitating vacuum. In accordance with (11) there are two types of solutions: with vanishing and not vanishing value of S_2. If $S_2 = 0$, then we have $S_1 = 0$ and according to cosmological equations in this case $H = 0$. Such solution corresponds to Minkowski spacetime in the vacuum. For the second solution with not vanishing value of S_2 we have:

$$4q_2S_1(3H - 4S_1) - \frac{q_1 + q_2}{3}F + 2(q_1 + q_2 - 2f)A_2 - (f_0 - b) = 0. \quad (12)$$

Eqs. (10) and (12) allow to determine the values of S_2^2 and H^2 for the vacuum as functions of available indefinite parameters in gravitational equations for HIM. This solution corresponds to 4-dimensional flat de Sitter spacetime with not vanishing torsion i.e. to the Riemann-Cartan continuum with constant curvature and torsion. It should be noted that the torsion in discussed vacuum solutions is connected with pseudoscalar torsion function S_2, and these solutions differ essentially from that obtained in the case of HIM with the only torsion function S_1 [19]. According to [19] in this case the vacuum solution with de Sitter metrics and not vanishing torsion is possible only if $a \neq 0$ [d]. As it is follows from our consideration, the regular character of the vacuum solutions obtained in this paper does not depend on restrictions on parameter a. In contrast to these vacuum solutions the vacuum solution in [19] is specific solution. In the frame of isotropic cosmology without higher derivatives ($a = 0$) based on gravitational Lagrangian (1) (see Refs. 7–15 and references herein) similar vacuum solutions do not appear. Because we have two possibilities for the vacuum – Minkowski spacetime and de Sitter spacetime with torsion, the following question appears: which of these possibilities is realized in nature (by assuming that the PGTG is correct gravitation theory). The answer to this question depends on the behaviour of cosmological solutions for HIM at asymptotics, when energy density of gravitating matter tends to zero. Mathematically the answer to the formulated question depends on restrictions on indefinite parameters in gravitational equations for HIM.

Now we will obtain the vacuum solution at some physically acceptable restrictions on indefinite parameters. In the frame of isotropic cosmology without higher derivatives, the absence of which is ensured by condition $a = 0$, equations for HIM include in general case four indefinite parameters (see for example [13]): $\alpha = \frac{f}{3f_0^2}$ with inverse dimension of energy density ($f > 0$), b with the same dimension as f_0 and

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[d]Such solution coincides with that obtained in [20] in the case $f_0 = 0$.

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two dimensionless parameters: $\varepsilon = \frac{q_2}{f}$ and $\omega = \frac{q_1 + q_2 - 2f}{f}$ [e]. By some restrictions on indefinite parameters the cosmological equations take at asymptpotics the form of Friedmann cosmological equations of GR with effective cosmological constant induced by spacetime torsion and allow to explain the acceleration of cosmological expansion at present epoch without using notions of dark energy and also dark matter (at least partially) [13]. The behaviour of cosmological solutions depends essentially on restrictions on parameters ε and ω. The most simple from mathematical point of view and physically acceptable case corresponds to the following choice: $\varepsilon = 0$ and $\omega \neq 0$. In this case the equations (12) and (10) lead to the following vacuum solutions for S_2^2 and H^2: $S_2^2 = \left[1 - \frac{b}{2f_0}[1 \pm \left(1 - \omega(1 - b/f_0)\frac{f_0^2}{b^2}\right)^{1/2}]\right][12\alpha b(1+\omega/4)]^{-1}$;

$H^2 = \frac{6b^2}{f_0}\alpha S_2^4[1 - 6\alpha(2b + \omega f_0)S_2^2]^{-1}$. The answer to the question "Which of these solutions corresponds to the true vacuum?" depends on additional restrictions on indefinite parameters and also on properties of equation of state of gravitating matter at the beginning of cosmological expansion, by which the cosmological equations lead to regular cosmological solutions with corresponding asymptotics.

4. Conclusion

In the framework of the standard gravitation theory (GR) de Sitter spacetime appears as a result of introducing of cosmological constant, which corresponds to some gravitating object with negative pressure, into Einstein gravitation equations [f]. By usual interpretation, the cosmological constant is associated with the vacuum of quantized matter fields. It should be noted that in the frame of quantum field theory the vacuum energy density of quantized fields diverges, and it can be eliminated by means of regularizing procedure. At the same time the value of cosmological constant in standard ΛCDM-model corresponds to very small energy density comparable with average energy density in the Universe at present epoch.

As it was shown in this paper, in the framework of PGTG de Sitter spacetime appears as a result of exact solution of gravitational equations for HIM with two torsion functions without cosmological constant. If the spacetime in the vacuum has the structure of de Sitter spacetime with torsion, then in the framework of classical field theory the conception of the vacuum as physical notion is changed essentially. Instead of vacuum as passive receptacle of physical objects and processes, the vacuum assumes a dynamical properties as a gravitating object. That leads to principal differences of gravitational interaction in comparison with other fundamental physical interactions, which are connected with certain matter properties and manifestation of which disappears without physical matter. In contrast to this, the vacuum possesses important characteristics of gravitating objects – the curvature and torsion.

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[e]The parameter ω was supposed to be equal to zero in [12,13,14].
[f]In the framework of PGTG de Sitter spacetime induced by cosmological constant was considered in [19].

If the spacetime of the vacuum is de Sitter spacetime with torsion, the acceleration of cosmological expansion at present epoch explained in the frame of PGTG in [12,15] acquires the vacuum origin. As it was noted in [13], the search for the criteria that allow us to be able to choose physically acceptable solutions is important for PGTG. According to [5] any vacuum solution of Einstein gravitation equations of GR (in particular, the Schwarzschild vacuum solution) with vanishing torsion is an exact solution of PGTG independently on values of indefinite parameters of gravitational Lagrangian (1). If the physical spacetime in the vacuum possesses the torsion, such solutions are not physically acceptable, and the search of corresponding physically acceptable solutions becomes well warranted. Because in the frame of PGTG Newton's law of gravitational interaction can be not applicable at cosmological and possibly astrophysical scales, approximative analysis of solutions of PGTG, in the frame of which Newton's law is used in the lowest approximation, has to be re-examined. It should be noted that the analysis of the particle content of PGTG based on general expression of gravitational Lagrangian in [5,21,22,23] is given in torsionless backgrounds – Minkowski spacetime and Einstein manifolds. If the physical vacuum has the structure of de Sitter spacetime with torsion, the investigation of the particle content of PGTG in such background is of certain interest.

We see that the Poincaré gauge theory of gravity leads to principal consequences concerning the classical notion of physical vacuum. By certain restrictions on indefinite parameters of gravitational Lagrangian (1), unlike some other generalizations of Einsteinian gravitation theory (see for example [25,26]) PGTG is free of such pathological objects as ghosts and tachyons, and cosmological solutions for accelerating Universe are asymptotically stable.

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